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M.A. (Mathematics) (Part-I) (External) Examination March - 2026

Mathematics - Graph Theory and Discrete Structure

Time: 3 Hours] **[Total Marks: 100**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book. (2) Attempt all questions. (3) Each questions carries equal marks. (4) Follow usual notations and conventions. (5) Use of non-programmable scientific calculator is permitted.	Student's Signature

ENGLISH VERSION

Q-1. a) i) Prove that a simple graph with n vertices and k components can have at most $(n - k)(n - k + 1)/2$ edges. **07**

ii) Prove that in a connected graph G with exactly $2k$ odd vertices, there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a Unicursal graph.

b) i) Prove that a tree with more than 2 vertices will always have at least 2 pendant vertices. **07**

ii) Prove that a graph is a tree if and only if it is minimally connected.

c) Discuss the Konigsberg Bridge problem. **06**

OR

Q-1. a) Using an example prove that union, intersection and ring sum of graphs is commutative. **07**

b) i) Show that the distance between vertices of a connected graph is a metric. **07**

ii) Prove that every tree has either one or two centres.

c) Discuss the Seating Problem. **06**

Q-2. a) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets **07**

b) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. **07**

c) Prove that $K_{3,3}$ is non-planar. **06**

OR

Q-2. a) Define: Spanning tree, Branch and Chord. **07**

Prove that any given edge of a connected graph G is a branch of some spanning tree. Is it also true that any arbitrary edge of G is a chord for some spanning tree of G?

b) Prove that in a complete graph with $n(\geq 3)$ vertices, there are $(n - 1)/2$ edge disjoint Hamiltonian circuits where n is an odd number. **07**

c) Prove that in a connected graph G, a vertex v is a cut-vertex if and only if there exist at least two edges x and y incident on v such that no circuit in G includes both x and y. **06**

Q-3. a) Discuss the observation about the cut-set matrix. **07**

b) i) Prove that any subgraph g of a connected graph G is contained in some spanning tree of G if and only if g contains no circuits. **07**

ii) Prove that a pendant edge in a connected graph G is contained in every spanning tree.

c) Define: Cute and Fundasental cut-set. **06**

Prove that a branch b_i, that determines a fundamental cut-set S, is contained in every Fundamental circuit associated with the chords of S and in no other.

OR

Q-3. a) State and prove Euler's formula. **07**

b) i) Prove that, no element of V^* is idempotent, except the empty string. **07**

ii) If g is a onto semigroup homomorphism from a semigroup $< S, *>$ to an algebraic structure $<T, \Delta>$, then prove that $<T, \Delta>$, is also a semigroup.

c) Define: Homomorphism Epimorphism and Monomorphism. **06**

Let f be a homomorphism from $< X, o>$ onto $<Y, \oplus>$, E be a congruence relation corresponding to f and g_E be the natural homomorphism from $<X, o>$ to $<\frac{X}{E}, *>$. Then prove that $h: \frac{X}{E} \rightarrow Y$ is an isomorphism, where $h[x] = f(x)$; $x \in X$.

Q-4. a) Discuss the application of Euler's theorem to find x. **07**

b) Prove that $< Z_m, +m >$ and $< Z^*_m, \oplus m >$ are isomorphic. **07**

c) For the set of natural numbers N, prove that $< N +>$ is a semigroup. Is the set of odd non-negative integers form a subsemigroup for $< N, +>$? Justify. **06**

OR

Q-4. a) i) Let g is a monoid homomorphism from $< M, *, e_m >$ to $< T, \Delta, e_T\} >$. Then prove that $g(a^{-1}) = [g(a)]^{-1}$; for all $a \in M$. **07**

ii) Prove that for a semigroup homomorphism commutativity is preserved.

b) State and prove the "Chinese remainder theorem". **07**

c) i) Prove that the rank of a well-formed polish notation is 1. **06**

ii) Write a note on number systems.

Q-5. a) Prove that if a and m are relatively prime integers then $a' = a^{f(m)-1} \bmod m$. **07**

b) State and prove the distributive inequality for a lattice. **07**

c) State and prove the isotonicity for a lattice. **06**

OR

Q-5. a) Prove that the operations of meet and join on a lattice is commutative, associative, idempotent and satisfies the law of absorption. **07**

b) Prove that for $S = (a, b, c) < \rho(S), \cap, \cup, ', \varnothing, S>$ is a Boolean algebra. **07**

c) i) In a lattice $< L, \leq >$, show that **06**

$(a * b) \oplus (c * d) \leq (a \oplus c) * (b \oplus d)$ and

$(a * b) \oplus (b * c) \oplus (c * a) \leq (a \oplus b) * (b \oplus c) * (c \oplus a)$

ii) Is $< S_{75}, D >$ a complemented lattice? Justify your answer.